Math 10A
Quiz 7; Tuesday, 7/24/2018
Time: 3 PM
Instructor: Roy Zhao **Name:** ____________________

Circle True or False. (1 point for correct answer, 0 if incorrect)

1. **TRUE** False The recurrence relation $a_n = 2a_{n-1} + 3a_{n-2} + n$ has constant coefficients.

   Solution: The coefficients of the a_i are $1, 2, 3$ which are numbers or constants.

2. **TRUE** False The recurrence relation $a_n = n^2 a_{n-2}$ is linear.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) For all three parts, consider the recurrence relation $a_n = (4-n)a_{n-1} + a_{n-2}^2$ with $a_0 = 0, a_1 = 1$.

   (a) (3 points) Calculate a_2, a_3, and a_4. Show your work.

   Solution: $a_2 = (4-2)a_1 + a_0^2 = 2 \cdot 1 + 0^2 = 2$. $a_3 = (4-3)a_2 + a_1^2 = 2 + 1^2 = 3$. $a_4 = (4-4)a_3 + a_2^2 = 2^2 = 4$.

   (b) (2 points) Find the order and determine whether it is homogeneous, linear and/or has constant coefficients. Justify all your answers.

   Solution: It has order 2 because the furthest back we go is a_{n-2}. It is homogeneous because there are no terms other than the a_i and their coefficients. It is not linear because a_{n-2} is squared. It does not have constant coefficients because of the coefficient $4-n$.

   (c) (5 points) Verify that $a_n = n$ is the solution to this recurrence problem.

   Solution: We need to plug in the initial conditions first to see if they work. They do because $a_0 = 0$ and $a_1 = 1$.
   Now we plug it into the relation to get $LHS = n$ and

   $$RHS = (4-n)(n-1) + (n-2)^2 = 4n - 4 - n^2 + n + n^2 - 4n + 4 = n = LHS.$$